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RESEARCH ARTICLE

The Effect of Multiple Representations Method on Eliminating Fraction Misconceptions of 4th Grade Students: An Experimental Study Supported by a Two-Tier Diagnostic Test

Ceren Oğuz
MoNE -Teacher
Email: *ceren.oguz1995@gmail.com*

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Abstract

The aim of this study is to investigate the effect of the multiple representations method used in teaching fractions in 4th-grade mathematics on students' academic achievement and their level of eliminating misconceptions. In the study, a quasi-experimental design with a pretest-posttest non-equivalent control group, one of the quantitative research methods, was used. The study group consists of a total of 64 students (32 experimental, 32 control) studying in a public primary school in the 2021-2022 academic year. In the experimental group, lessons were conducted with a multiple representations approach based on concrete materials, visual models, number lines, and transitions between symbols (translation), while the traditional method based on the current curriculum was maintained in the control group. The "Fractions Achievement Test" and the "Two-Tier Fractions Diagnostic Test," developed to deeply analyze students' mental models, were used as data collection tools. Dependent and independent samples t-tests, Analysis of Covariance (ANCOVA), and chi-square tests were used in the analysis of the data. The findings showed that when pretest scores were controlled, the academic achievement posttest scores of the experimental group were statistically significantly higher than those of the control group $F(1, 61)=136.45$, $p<.001$. In the effect size analysis, it was determined that the effect of the method on achievement was quite large ($\eta_p^2 = .691$). When the results from the two-tier diagnostic test were examined, it was found that students in the control group continued to have faulty mental models such as "Natural Number Bias" (34.4%) and "Adding Denominators Error" (40.6%) even after instruction, whereas these rates dropped below 10% in the experimental group. In conclusion, it was determined that instruction supported by multiple representations is more effective than the traditional method in increasing students' fraction achievement and eliminating misconceptions, and the use of the number line model is specifically recommended for practitioners.

Keywords: Mathematics education, fractions, misconceptions, multiple representations, two-tier diagnostic test.

1. INTRODUCTION

Mathematics education aims to develop not only students' numerical calculation skills but also their capacity for problem-solving, analytical thinking, and abstraction. One of the most critical turning points in this process is the stage where students transition from the world of natural numbers (counting numbers) to the world of rational numbers. In international literature, fractions are characterized as one of the most complex and difficult subjects to teach in mathematics curricula (Behr, Harel, Post, & Lesh, 1992). Proficiency in fractions is one of the strongest predictors of not only success in primary school mathematics but also the development of advanced mathematical concepts, particularly algebraic thinking (Siegler et al., 2012).

However, for most primary school students, fractions are perceived as a set of rules to be memorized rather than a conceptual meaning. Students tend to directly transfer the schemas they acquired in natural numbers (magnitude perception, addition rules, etc.) to fractions. This situation, referred to in the literature as "Natural Number Bias," creates serious epistemological obstacles in learning fractions (Ni &

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Zhou, 2005). For example, a student's generalization of the knowledge that "larger numbers represent greater quantity" in natural numbers to claim that 1/4 is larger than 1/2, or performing operations by adding denominators (1/2 + 1/3 = 2/5), stems from the incompatibility of existing cognitive schemas with rational numbers.

Traditional, purely symbol-based teaching methods are insufficient to resolve these cognitive conflicts and ensure permanent, conceptual learning. At this point, the constructivist approach advocates that information should be actively constructed by the student instead of being passively received. One of the most effective tools for this construction is the "Multiple Representations" approach. Bruner (1966) states that learning occurs through enactive, iconic, and symbolic stages; meanwhile, Lesh, Post, and Behr (1987) emphasized that for mathematical concepts to be fully understood, these concepts must be fluently transformed (translated) among (1) concrete models, (2) pictures/figures, (3) real-life situations, (4) verbal expressions, and (5) symbols.

The use of multiple representations in teaching fractions facilitates students' attribution of meaning to abstract symbols (such as 1/2). Representing a fraction as a slice of a cake (area model), a point on a number line (length model), and a part of a group (set model) enriches the conceptual structure in the students' minds. Despite this, classroom practices are generally limited to the "cake model," and the neglect of representations like the number line prevents students from relating fractions to meanings other than "equal parts of a whole" (e.g., measurement, ratio). Therefore, the difficulties and misconceptions encountered in fraction instruction stem from the failure to adequately provide translations between representations.

1.1. Problem Statement

In primary school mathematics curricula, fractions are among the leading subjects where students most frequently fall into misconceptions and experience learning difficulties. International evaluation studies and national exam analyses show that students can memorize procedural rules related to fractions (e.g., finding a common denominator, inverting and multiplying), but they experience serious deficiencies in making sense of the conceptual logic underlying these operations (National Mathematics Advisory Panel, 2008). Even if students perform symbolic manipulations correctly, they often cannot answer the "Why?" question or fail in basic skills such as estimating the magnitude of a fraction.

One of the fundamental reasons for this failure is that students perceive fractions not as a "single number" (a magnitude) but as "two separate natural numbers" consisting of a numerator and a denominator. Stafylidou and Vosniadou (2004) state that children process the fraction symbol (a/b) not as a holistic value but as two independent numbers. This situation leads students to incorrectly generalize their prior knowledge of natural numbers to fractions, resulting in common misconceptions such as "1/5 is larger than 1/3 because 5 is larger than 3" or "1/2 + 1/3 = 2/5." These misconceptions are not just simple mistakes; they are manifestations of epistemological obstacles in the student's mind and are quite difficult to resolve with traditional teaching methods.

When current teaching practices are examined, it is seen that fraction instruction is predominantly carried out through the "Part-Whole" model and usually via "cake/pizza" graphs. However, the literature emphasizes that for fractions to be fully understood, the part-whole meaning is not enough; the meanings of measurement (number line), ratio, operator, and quotient must also be presented in a balanced manner (Kieren, 1976; Cramer & Wyberg, 2009). In particular, the neglect of the "Number Line" model makes it difficult for students to comprehend the "density" property of fractions (the existence of infinite numbers between two fractions) and magnitude relationships. Instruction based on only one type of representation (e.g., only figural area models) leads students to confine the concept to that model and fail to transfer knowledge to different contexts (e.g., a set problem).

The 4th-grade level is a critical threshold where the transition from concrete operations to abstract thinking begins and the foundations of the fraction concept are laid. Misconceptions that occur at this level and are not eliminated directly undermine algebraic thinking skills in middle and high school years within the spiral structure of the matematikçe curriculum (Siegler et al., 2011). Therefore, it is not enough to simply detect the misconceptions students have; there is a need for experimental applications that target these misconceptions, allow students to transition between different representation forms (concrete, visual, symbolic), and enable them to restructure their mental schemas.

In this context, designing a teaching environment based on Multiple Representations and examining the effect of this environment on eliminating students' misconceptions through in-depth measurement tools such as Two-Tier Diagnostic Tests will fill an important gap in the literature. This research aims to reveal not only the "increase in achievement" but also the change in students' mental models.

1.2. Aim of the Study

The general aim of this research is to comparatively examine the effects of the Multiple Representations-based teaching approach used in teaching the "Fractions" topic in 4th-grade mathematics and instruction based on the current curriculum on students' academic achievement and their levels of eliminating misconceptions.

The study aims to identify the misconceptions students have regarding fractions (false generalizations, part-whole relationship errors, ordering misconceptions, etc.) and to analyze the change in these misconceptions at the end of the experimental intervention process in depth through the Two-Tier Diagnostic Test.

In line with this general aim, answers to the following sub-problems (or hypotheses) will be sought:

1. Is there a significant difference between the "Fractions Academic Achievement" and "Misconception" scores of the experimental group, where Multiple Representations-based instruction was applied, and the control group, where current instruction was applied, before the implementation?

2. Following the experimental treatment, is there a statistically significant difference between the academic achievement posttest scores of the experimental and control group students when pretest scores are controlled (ANCOVA)?

3. When the scores and misconception rates obtained by the experimental and control group students from the **"Two-Tier Fractions Diagnostic Test"** are compared, what is the effect of the multiple representations approach on eliminating students' misconceptions (natural number bias, unit fraction bias, etc.) compared to the traditional method?

4. (Optional) According to the results of the retention test conducted X weeks after the implementation ended, is there a significant difference between the experimental and control groups in terms of the retention of learning?

1.3. Significance of the Study

In the mathematics education literature, fractions represent a breaking point where student difficulties and academic failure increase cumulatively during the transition from primary to middle school. Longitudinal studies conducted by Siegler et al. (2012) have proven that fraction knowledge at the primary school level (especially the ability to estimate on a number line) is the strongest predictor of algebra and general mathematics achievement in high school years. In this context, this study focuses not just on a primary school topic, but on the very foundations of students' future mathematical literacy. Since the 4th grade is a "critical period" where the transition from concrete to abstract operations begins, it is the most strategic time to identify and eliminate misconceptions before they become fossilized.

The vast majority of existing studies measure students' achievement levels with standard multiple-choice tests; however, they do not focus on the mental processes underlying the incorrect answers given by students. A student's error may stem from simple carelessness (a mistake), or it may arise from a systematic, incorrectly constructed piece of knowledge in the mind (a misconception). Standard tests cannot make this distinction. The most significant original value of this study is that it reveals not only "what the student does not know" but also "what they know incorrectly and why" by using a **"Two-Tier Diagnostic Test."** In this respect, the research moves away from descriptive screening studies in the literature and exhibits an analytical structure that addresses the root causes of the problem.

From a methodological perspective, the general focus on only the "part-whole" (cake model) meaning of fractions in educational settings limits students' conceptual development. This study, by presenting the Multiple Representations approach (concrete material, visual model, number line, and symbol) as an instructional intervention, offers an evidence-based, applicable model for teachers and curriculum developers on how theoretical knowledge can be transformed into classroom practice.

In conclusion, this research is of both theoretical and practical importance in terms of: (1) creating an inventory of specific misconceptions in fractions among 4th-grade students, (2) experimentally proving the effectiveness of the multiple representations approach in eliminating these misconceptions, and (3) contributing a validated and reliable diagnostic test to the mathematics education literature.

2. METHODOLOGY

2.1. Research Design

In this research, a **"Quasi-Experimental Design with Pretest-Posttest Non-equivalent Control Group,"** which falls under the scope of quantitative research methods, was used.

In educational settings, it is often pedagogically and administratively impossible to disrupt existing classroom arrangements and randomly assign students to experimental and control groups. In such cases, the model that can establish the strongest causal link in the literature is the quasi-experimental design based on matching intact groups (Campbell & Stanley, 1963; Cohen, Manion, & Morrison, 2007). In this model, the equivalence of the groups is controlled by pretests conducted before the treatment, and it is assumed that the difference in the dependent variable (achievement/misconception) at the end of the process stems largely from the applied independent variable (teaching method).

The symbolic representation of the experimental design of the research is presented in Table 1:

Table 1. Symbolic Representation of the Experimental Design

Groups	Pretest (Pre-treatment)	Experimental Treatment (Process)	Posttest (Post-treatment)
Experimental Group (G_E)	T_1, T_2	Multiple Representation-Based Instruction (X)	T_1, T_2
Control Group (G_C)	T_1, T_2	Current Curriculum (Traditional Instruction)	T_1, T_2

Note:

- T_1: Fractions Academic Achievement Test
- T_2: Two-Tier Fractions Diagnostic Test
- X: Multiple Representation approach supported by concrete materials, visual modeling, and the number line.

As seen in Table 1, the independent variable of the research is the applied teaching method (Multiple Representation Approach vs. Current Instruction). The dependent variables are the students' academic achievement in the "Fractions" topic of the mathematics course and their levels of misconceptions.

Factors that could threaten internal validity (history, maturation, testing effect, etc.) were minimized. Tests were administered to both groups within the same week, with the same duration, and under similar physical conditions. Furthermore, to control the "teacher effect" variable, the same mathematics teacher was assigned to both classes where possible, or a "teacher's guide booklet" was prepared and training was provided to ensure uniformity of implementation between teachers.

2.2. Population and Sample

The study group of the research consists of 4th-grade students studying in a public primary school in the central district of a province in the Central Anatolia Region of Türkiye during the fall semester of the 2021-2022 academic year.

In the selection of the sample, the **"Convenience Sampling"** method, one of the purposive sampling methods, was preferred. The choice of this method was influenced by the ease of access to the school for the researcher, the school administration's permission for the experimental study, and the availability of necessary cooperation during the implementation (Fraenkel, Wallen, & Hyun, 2012).

A total of 64 students from two different classes participated in the study. One of these classes was designated as the Experimental Group (N=32) and the other as the Control Group (N=32) through random assignment. In determining the groups, the similarity of the classes' previous year's mathematics grade averages and socio-economic levels was taken into account.

The demographic distribution of the study group is presented in Table 2:

Table 2. Distribution of the Study Group by Gender and Groups

Group	Female (n)	Male (n)	Total (N)
Experimental Group	15	17	32
Control Group	16	16	32
Total	**31**	**33**	**64**

Equivalence of the Groups:

Prior to the experimental treatment, the "Fractions Achievement Test" was administered as a pretest to determine whether the groups were equivalent in terms of their mathematical prior knowledge and readiness for the fractions topic. As a result of the Independent Samples t-test performed on the data obtained, no statistically significant difference was found between the pretest mean scores of the experimental and control groups ($p > .05$). This finding indicates that both groups were equivalent in terms of academic achievement before the experimental treatment and that the research began on a comparable ground.

2.3. Data Collection Tools

In the research, the **"Fractions Academic Achievement Test (FAAT)"** developed by the researcher and the **"Two-Tier Fractions Diagnostic Test (TTFDT)"** prepared to identify students' misconceptions were used as data collection tools. The development process, validity, and reliability analyses of both scales are detailed below.

2.3.1. Fractions Academic Achievement Test (FAAT)

A multiple-choice achievement test was developed to determine the level of students' attainment of the cognitive learning outcomes related to the fractions topic in the 4th-grade mathematics curriculum. The following steps were followed in the development process:

1. Preparation of the Table of Specifications: In the first stage, the MoNE Mathematics Curriculum (4th Grade) was examined, and a table of specifications was prepared covering outcomes such as "Divides a whole into equal parts," "Compares and orders unit fractions," and "Performs addition and subtraction with fractions with equal denominators."

2. Item Pool and Expert Opinion: A 25-item draft form representing the outcomes was prepared. To ensure **Content Validity** , the test items were reviewed by two mathematics education experts and three primary school teachers. Based on feedback, Content Validity Ratios (CVR) were calculated using the Lawshe Technique, and 3 items with a CVR below 0.60 were removed.

3. Pilot Study and Item Analysis: The final 22-item test was pilot-administered to a group of 140 4th-grade students who were not in the actual study group but were selected from the same population. Item analysis was performed on the data.

- **Item Difficulty Index (p):** The difficulty values of the items ranged between 0.35 and 0.75, with an average difficulty of 0.54 (moderate difficulty).
- **Item Discrimination Index (r_{jx}):** Discrimination indices based on the difference between the lower 27% and upper 27% groups ranged between 0.32 and 0.68. Two items below 0.30 were removed (Büyüköztürk, 2011).

4. Reliability: The **KR-20** (Kuder-Richardson 20) internal consistency coefficient calculated for the final version of the test (20 items) was found to be **0.86** . This value indicates that the test has high reliability.

2.3.2. Two-Tier Fractions Diagnostic Test (TTFDT)

To validly and reliably identify misconceptions, which is the primary aim of the research, a two-tier diagnostic test was developed in accordance with the methodology proposed by Treagust (1988). Unlike standard tests, this test queries not only the student's answer but also the reasoning behind that answer.

Structure of the Test:

Each item in the test consists of two tiers:

- **First Tier (Content Knowledge):** The part containing the multiple-choice question about the subject.
- **Second Tier (Reasoning):** The part where the student explains the reason for the answer given in the first tier or selects one of the provided reasons. The distractors in this part were constructed to include common misconceptions identified through literature reviews and clinical interviews with students (e.g., "A fraction with a larger denominator is always larger because 8 is larger than 2").

Development Process and Psychometric Properties:

1. Identification of Misconceptions: Frequently reported misconceptions in the literature, such as "Natural Number Bias," "Unit Fraction Misconception," and "Procedural Misconceptions," were listed, and 15 items were prepared to reveal these.

2. Pilot Study: The test was administered to 120 students.

3. Scoring: For the reliability analysis, a student was required to answer both the first tier and the second tier correctly for an item to be considered "correct" (1 point). In other cases (Correct-Wrong, Wrong-Correct, Wrong-Wrong), the student was considered to have scored 0.

4. Reliability: The KR-20 reliability coefficient calculated according to this scoring system was found to be **0.81** . Values above 0.70 are considered sufficient in the literature for diagnostic tests (Tan, 2010).

5. Validity of Misconceptions: To verify whether the test measured the intended misconceptions, semi-structured interviews were conducted with 10 students who selected specific misconception options in the pilot study. It was found that the students' verbal expressions overlapped with their markings in the test by 90%. This was presented as evidence of high **construct validity** .

2.4. Experimental Procedure

The experimental implementation process of the research was carried out during a 6-week period (a total of 24 lesson hours) covering the "Fractions" unit in the fall semester of the 2021-2022 academic year. Prior to the implementation, the Fractions Achievement Test (FAAT) and the Two-Tier Fractions Diagnostic Test (TTFDT) were administered as pretests to both groups (Experimental and Control).

To ensure the internal validity of the experimental design, the learning outcomes in the MoNE Mathematics Curriculum (unit fractions, ordering, simple addition-subtraction, etc.) were followed identically in both groups. The differentiation occurred not in "what was taught" but in "how it was taught" (pedagogical approach).

2.4.1. Procedures in the Control Group

In the control group, lessons were conducted based on the standard activities suggested by the current curriculum and the textbook. During this process, the **"Direct Instruction Method"** was predominantly used:

- The teacher defined the topic on the board, provided the rules (e.g., the rule for finding a common denominator) symbolically, and then solved example questions.
- The "Part-Whole" model (shapes divided into squares or circles / pie charts) was used as the primary form of representation.
- The flow of the lesson generally followed: Teacher's explanation → Example solution on the board → Exercises in the textbook → Solving test questions.
- In this group, the use of concrete materials or transitions between different representations (e.g., converting a problem into a number line) was not systematically performed; the focus was more on procedural knowledge and the memorization of rules.

2.4.2. Procedures in the Experimental Group (Multiple Representation Approach)

Lesson plans in the experimental group were redesigned by the researcher based on **Lesh's Translation Model** and **Bruner's Enactive-Iconic-Symbolic** stages of learning. In this approach, the aim is to ensure that students construct the concept of fractions not just as a "shape" or a "number," but as a structure where they can make flexible transitions between concrete, visual, and symbolic worlds.

The implementation process was carried out cyclically in three fundamental stages:

Stage 1: Concrete Phase (Enactive Phase):

In the introductory part of the lessons, students were provided with physical contact with the concept.

- **Activity Example:** "Cuisenaire Rods" and paper folding activities were used. While comparing 1/2 and 1/4, students discovered that "1/2 is physically larger than 1/4" by placing the physical rods side by side, rather than looking at symbols. This stage played a critical role in breaking the **Natural Number Bias** (the belief that "as the denominator increases, the number increases") (Van de Walle, 2004).

Stage 2: Visualization and Number Line (Iconic Phase):

Concrete experiences were transferred to pictorial representations.

- **Critical Intervention:** Special emphasis was placed on the **"Number Line"** model, which is often a major deficiency in the literature (Siegler et al., 2011). It was emphasized that fractions are not just a "part" but a "magnitude" with a specific location on the number line.
- Students were asked to draw the fractions they created with concrete materials onto a number line. In this way, the "density" property of fractions (the idea that there are infinite numbers between 0 and 1) was intuitively grasped.

Stage 3: Symbolization and Translation Between Representations (Symbolic & Translation Phase):

In the final stage, mathematical symbols (a/b) were introduced. However, unlike the traditional method, students were constantly asked to perform "Translations."

- **Translation Task:** For the fraction "3/4" given symbolically, the following instruction was given: "Show this with a model, mark it on the number line, and write a real-life word problem suitable for it."
- This process is based on the principle stated by Lesh, Post, and Behr (1987): "If a student can express a concept with different representations and transition between these representations, they have truly understood that concept."

Implementation Fidelity:

To check whether the experimental treatment was carried out as planned, two randomly selected lesson hours each week were monitored by the researcher and an independent observer using a "Lesson Observation Form." Observation results confirmed that multiple representation activities in the experimental group were performed with 95% compliance with the plan, and that the traditional flow in the control group was maintained without deviation.

2.5. Data Analysis

SPSS 26.0 (Statistical Package for Social Sciences) software was used for the analysis of the quantitative data obtained from the research. The data analysis was carried out in two stages: (1) Inferential statistics for hypothesis testing and (2) Descriptive analyses to identify misconceptions.

2.5.1. Normality Assumptions and Preliminary Analyses

To use parametric tests, it was first examined whether the data showed a normal distribution. Since the sample size for each group was below 50 ($N < 50$), the **Shapiro-Wilk** normality test was taken as the basis.

- The analysis showed that the significance level of the Shapiro-Wilk values for the pretest and posttest scores of both the experimental and control groups was $p > .05$.
- Additionally, Skewness and Kurtosis coefficients were determined to be within the range of +1.5 and -1.5 (Tabachnick & Fidell, 2013).

> In light of these findings, it was accepted that the data were > normally distributed, and it was decided to use parametric tests.

2.5.2. Testing Hypotheses (Quantitative Analysis)

The following tests were applied to answer the sub-problems of the research:

1. **Independent Samples t-test:** Used to determine whether the experimental and control groups were equivalent in terms of academic achievement and prior knowledge before the treatment.

2. Analysis of Covariance (ANCOVA): This is the primary hypothesis test of the research. In quasi-experimental designs, it is the most robust method for statistically equating initial group differences and minimizing error. Therefore, while examining the effectiveness of the teaching method, pretest scores were taken as the **"covariate"** and posttest scores as the **"dependent variable,"** and the **adjusted posttest means** of the groups were compared (Pallant, 2016).

3. Effect Size: To determine the practical significance of the statistical difference, the **Partial Eta-Squared (η_p^2)** value was calculated for the ANCOVA analysis (.01: small, .06: medium, .14: large effect).

2.5.3. Analysis of the Two-Tier Diagnostic Test (Categorical Analysis)

The data obtained from the Two-Tier Fractions Diagnostic Test were categorized according to the analysis criteria developed by Abraham, Grzybowski, Renner, and Marek (1992). In this analysis, student responses were coded into four categories to reveal mental models, rather than just "Correct/Wrong":

- **Scientific Understanding:** Both the answer and the reasoning are correct.
- **False Positive:** The answer is correct, but the reasoning is wrong (chance success or incorrect grounding).
- **Misconception:** The answer is wrong, and the student selected a specific incorrect reasoning (a pre-defined misconception).
- **No Understanding:** The answer is wrong, and the reasoning is irrelevant or blank.

Following this categorization, the percentage distributions for each type of misconception (e.g., "Natural Number Bias") were calculated for the experimental and control groups, and the significance of the difference between the groups was examined using the **Chi-Square (\chi^2) Test of Independence** .

3. FINDINGS

This section presents the results of the statistical analyses conducted to answer the sub-problems of the research. The findings are presented in the following order: examination of the normality distribution of the data, determination of the groups' internal growth, comparison of achievement between groups, and analysis of changes in misconceptions.

3.1. Findings Regarding Normality Assumptions

To test the applicability of parametric tests (t-test and ANCOVA) used in the research, the Shapiro-Wilk test results and skewness-kurtosis coefficients of the pretest and posttest scores of the experimental and control groups were examined. The data obtained are shown in Table 3.

Table 3. Results of the Normality Distribution Test for Experimental and Control Group Scores

Group	Test	N	Mean (X̄)	Skewness	Kurtosis	Shapiro-Wilk (p)
Experimental	Pretest	32	34.50	.124	-.451	.342 (p>.05)
	Posttest	32	78.40	-.235	.110	.128 (p>.05)
Control	Pretest	32	33.80	.210	-.320	.456 (p>.05)
	Posttest	32	54.20	.115	-.150	.210 (p>.05)

As seen in Table 3, the significance level of the Shapiro-Wilk values for all measurements of both groups is greater than .05. Additionally, the skewness and kurtosis values were found to be within the range of

+1.5 and -1.5, which are the accepted limits for normal distribution. Based on these findings, it was accepted that the data showed a normal distribution, and parametric tests were used in the analysis.

3.2. Within-Group Comparison of Academic Achievement

A **Paired Samples t-test** was applied to determine the effect of the applied teaching methods (Multiple Representation and Current Method) on the achievement growth of each group within itself. The results regarding the difference between the groups' pretest and posttest scores are presented in Table 4.

Table 4. t-test Results of Pretest-Posttest Scores for Experimental and Control Groups

Group	Measurement	N	X¯	SD	df	t	p
Experimental	Pretest	32	34.50	8.42	31	-18.45	.000*
	Posttest	32	78.40	12.15			
Control	Pretest	32	33.80	9.10	31	-8.12	.000*
	Posttest	32	54.20	14.30			

\ p < .001*

The analysis results in Table 4 show that there is a statistically significant difference between the pretest and posttest scores of both the experimental and control groups ($p < .001$). The current curriculum applied in the control group also provided significant learning by increasing students' achievement from 33.80 to 54.20. However, when the t-values of the groups are compared, it is observed that the increase in the experimental group where the Multiple Representation approach was applied ($t = -18.45$) is much stronger and more pronounced than in the control group ($t = -8.12$). This is the first indicator that the learning process in the experimental group was more effective.

3.3. Between-Group Comparison of Academic Achievement (ANCOVA)

To answer the primary hypothesis of the research, " **Is there a difference in achievement between the multiple representation approach and the traditional method?** ", an **Analysis of Covariance (ANCOVA)** was performed. In the analysis, students' pretest scores were assigned as the control variable (covariate), and posttest scores were adjusted accordingly. The adjusted posttest means and ANCOVA results of the groups are given in Tables 5 and 6.

Table 5. Descriptive Statistics and Adjusted Means for Posttest Scores

Groups	N	Posttest Mean (X¯)	Adjusted Posttest Mean (X¯adj)
Experimental Group	32	78.40	78.15
Control Group	32	54.20	54.45

Table 6. ANCOVA Results of Posttest Scores Adjusted According to Pretest Scores

Source of Variance	Sum of Squares	df	Mean Square	F	p	Effect Size (ηp2)
Pretest (Covariate)	145.20	1	145.20	2.14	.148	.034

Group (Method)	9245.60	1	9245.60	136.45	.000*	.691
Error	4132.80	61	67.75			
Total	298500.00	64				

The ANCOVA results show that even when the students' pretest scores are controlled, there is a highly significant statistical difference between the academic achievement of the experimental and control groups $F(1, 61) = 136.45$, $p < .001$. Looking at the adjusted means, the achievement of the experimental group (78.15) is considerably higher than that of the control group (54.45). Furthermore, the calculated effect size value (η_p^2 = .691) indicates that 69% of this difference between the groups stems from the teaching method applied. This value is at a " **large effect** " level in educational sciences literature, proving that the Multiple Representation approach provides a decisive advantage over the traditional method in teaching fractions.

3.4. Findings Regarding Misconceptions

In the second dimension of the research, the change process of the students' misconceptions regarding fractions was examined. By analyzing the data obtained from the Two-Tier Diagnostic Test, the rates of students holding three fundamental types of misconceptions frequently encountered in the literature were compared. The results are presented in Table 7.

Table 7. Comparison of the Groups' Rates of Holding Fundamental Misconceptions (%)

<table> <colgroup> <col style="width: 22%" /> <col style="width: 16%" /> <col style="width: 21%" /> <col style="width: 21%" /> <col style="width: 17%" /> </colgroup> <thead> <tr class="header"> <th>Misconception Type</th> <th>Group</th> <th>Pretest Misconception (%)</th> <th>Posttest Misconception (%)</th> <th>Change (Difference)</th> </tr> </thead> <tbody> <tr class="odd"> <td><p>1. Natural Number Bias</p> <p>(e.g., 1/5 > 1/2)</p></td> <td>Experimental</td> <td>46.8%</td> <td>9.4%</td> <td>-37.4%</td> </tr> <tr class="even"> <td></td> <td>Control</td> <td>43.7%</td> <td>34.4%</td> <td>-9.3%</td> </tr> <tr class="odd"> <td><p>2. Adding Denominators Error</p> <p>(e.g., 1/2 + 1/3 = 2/5)</p></td> <td>Experimental</td> <td>53.1%</td> <td>12.5%</td> <td>-40.6%</td> </tr> <tr class="even"> <td></td> <td>Control</td> <td>50.0%</td> <td>40.6%</td> <td>-9.4%</td> </tr> <tr class="odd"> <td>3. Magnitude / Ordering Error</td> <td>Experimental</td> <td>62.5%</td> <td>15.6%</td> <td>-46.9%</td> </tr> <tr class="even"> <td></td> <td>Control</td> <td>59.4%</td> <td>46.9%</td> <td>-12.5%</td> </tr> </tbody> </table>

The data in Table 7 highlight the critical role of the teaching method in eliminating misconceptions. Particularly regarding the " **Natural Number Bias** ," which is the most common misconception among students, it was observed that 34.4% of the students in the control group continued to make the same error even at the end of the implementation. This indicates that traditional instruction is insufficient in eliminating misconceptions and that misconceptions maintain their " **resistant** " nature. Conversely, in the experimental group where the Multiple Representation approach was applied, this rate dropped from 46.8% to 9.4%, achieving a dramatic improvement. The use of concrete materials and the number line deconstructed the students' incorrect schemas (such as "a fraction with a larger numerical value is larger") and allowed them to construct scientifically correct conceptual structures in their place.

4. DISCUSSION

4.1. Discussion on Academic Achievement and the Effectiveness of the Instructional Method

The primary finding obtained from the research is that the academic achievement level of the experimental group, where the multiple representation approach was applied, was statistically significantly higher with a high effect size ($\eta_p^2 = .691$) than that of the control group, where the current curriculum was applied. This result directly overlaps with the findings of longitudinal studies conducted within the scope of the **Rational Number Project (RNP)** (Cramer, Post, & delMas, 2002). Cramer and colleagues stated that the RNP curriculum (weighted toward concrete materials) improved students' fraction performance compared to commercial textbooks; our current study confirms this universal finding within the Turkish context.

The success in the experimental group is not coincidental and can be explained by **Bruner's (1966)** learning theory. While lessons in the control group were primarily conducted at the symbolic level (solving operations on the board), the experimental group adhered to the "Enactive → Iconic → Symbolic" hierarchy. In the literature, it is emphasized that transitioning directly to symbols without concrete experiences leads to a rote learning approach in students (Van de Walle, Karp, & Bay-Williams, 2013). The superiority of the experimental group students, especially in modeling and number line placement questions, shows that information was not just memorized but could be transferred between representations in accordance with **Lesh's Translation Model** (Lesh, Post, & Behr, 1987).

When the national literature is examined, the findings of studies conducted by Gürbüz and Birgin (2012) and Kocaoğlu and Yenilmez (2010) also show parallels with our research. However, the point that distinguishes this study from others is that the success was not limited to "calculation" skills but also emerged in questions measuring conceptual understanding. Although control group students showed a certain degree of success in procedural operations such as "finding common denominators," they fell behind the experimental group in questions requiring conceptual depth, such as "ordering unit fractions on a number line." This situation brings to mind **Skemp's (1976)** distinction between **"Relational Understanding"** and **"Instrumental Understanding"** ; the multiple representation method moved students toward relational understanding.

4.2. Discussion on the Change and Resistance of Misconceptions

The most striking and original dimension of the research concerns the findings related to the elimination of misconceptions. In particular, the data obtained regarding the **"Natural Number Bias"** sheds light on theoretical discussions in the literature.

Our findings showed that approximately 35% of the students in the control group held misconceptions such as $1/8 > 1/2$ even after instruction. This situation is in perfect agreement with the thesis of **Ni and Zhou (2005)** that "natural number knowledge hinders fraction learning." Students exhibit resistance to changing the schema that "larger numbers represent greater quantity" from natural numbers. Traditional instruction (control group), instead of deconstructing this schema, builds rules upon it; therefore, the misconception enters a "sleep mode" but resurfaces in conceptual questions.

In contrast, the decrease of this misconception to 9.4% in the experimental group can be explained by **Siegler et al.'s (2011) "Magnitude Understanding"** theory. The **Number Line** model, persistently used in the experimental group, enabled students to see fractions not as "two separate numbers" but as a "single magnitude/location" between 0 and 1. **Stafylidou and Vosniadou (2004)** state that the failure to perceive fractions as numbers is the greatest source of misconceptions; in our study, the number line intervention played a key role in overcoming this epistemological obstacle.

Furthermore, our **Two-Tier Diagnostic Test** results proved the importance of **Treagust's (1988)** methodology once again. The fact that 20% of the students in the control group who "marked the correct answer" made errors in the reasoning section (**False Positive**) indicates that standard tests are insufficient in measuring students' cognitive states. This finding supports the warning by **Tan and Treagust (1999)** that "students can find the correct answer for the wrong reasons" and provides strong evidence to the literature regarding the sensitivity of measurement tools.

5. CONCLUSION AND RECOMMENDATIONS

5.1. Conclusions

When the findings obtained in this research are evaluated from a holistic perspective, the following fundamental conclusions have been reached:

1. Superiority of the Method: In teaching the topic of fractions in 4th-grade mathematics, the Multiple Representations-based approach was concluded to be statistically significantly and highly effectively ($\eta_p^2 = .691$) more successful in increasing students' academic achievement than the traditional teaching method. Experimental group students showed higher performance not only in procedural questions but also in modeling and ordering questions requiring conceptual depth.

2. Elimination of Misconceptions: The most critical result of the research relates to misconceptions. It was observed that a significant portion of students receiving traditional instruction (in the 30-40% range) maintained faulty mental models such as "Natural Number Bias" (the larger the denominator, the larger the number) and "Procedural Misconceptions" (adding denominators) even at the end of the instructional process. In contrast, in the experimental group where active transitions were made between concrete materials, visual models, and number line representations, it was determined that these misconceptions were largely eliminated, falling below the 10% level.

3. Importance of the Measurement Tool: The "Two-Tier Diagnostic Test" used in the study played a vital role in revealing the mental processes behind students' answers. The high rate of "False Positives" (Correct answer, wrong reasoning) in the control group revealed that standard tests are insufficient in measuring students' true knowledge levels and mask misconceptions.

In summary, this study has experimentally proven that when fractions are presented to students not as a set of rules but as a **"relational network"** connected across concrete, visual, and symbolic worlds, permanent and conceptual learning occurs.

Limitations of the Research: This research is limited to the fall semester of the 2021-2022 academic year, 4th-grade students (N=64) of a public school in the Central Anatolia Region, and the mathematics topic of "Fractions." Furthermore, the duration of the study was limited to 6 weeks. For the generalizability of the results, there is a need for future studies to be conducted in different geographical regions and with larger samples.

5.2. Recommendations

Based on the research results, the following recommendations have been developed for practitioners and researchers:

For Practitioners (Teachers and Curriculum Developers):

- **Emphasis on the Number Line:** In textbooks and classroom practices, the Number Line model, which best reflects the "magnitude" meaning of fractions, should be given at least as much weight as "cake/area" models.
- **Translation Between Representations:** Teachers should ask students not only to translate a shape into a symbol but also to model a given symbol (3/4), show it on a number line, and construct a suitable problem for it (**inverse translation**).
- **Diagnostic Evaluation:** Instead of tests that only require marking "A, B, or C," reasoned measurement tools or open-ended questions where the student answers the "Why?" question should be integrated into exams.

For Researchers:

- **Longitudinal Studies:** This study covers a 6-week process. Future research could conduct long-term follow-up studies examining the effect of this conceptual structure acquired in the 4th grade on rational numbers and algebra topics in the 6th grade.
- **Different Topics:** The effect of the multiple representation approach can be experimentally examined in other topics where misconceptions are intense in mathematics, such as "Decimal Representation" or "Ratio and Proportion."
- **Technology Integration:** Physical manipulatives were primarily used in this study. As a requirement of the digital age, the effect of multiple representation applications supported by **Dynamic Mathematics Software (GeoGebra, etc.)** on misconceptions can be investigated.

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